How far can learning with representations and reasoning go?
Examples with quadratic functions

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Who we are …

- Ann – original background in mathematics
  - Experienced high school teacher
  - Instructor of teacher candidates for more than 20 years
  - Researcher in mathematics for teachers

- Jennifer – original background in education
  - Experienced elementary school teacher
  - Instructor of teacher candidates for more than 10 years
  - Researcher in mathematics for teachers

Our context

- Education and the school curriculum in Canada is provincially mandated, so we refer here to our own province, Ontario
- Elementary (grades 1-8) curriculum revised in 2020
- Grade 9 curriculum revised and de-streamed in 2021
- Grades 10-12 pending revision
- Teacher education is a two-year post-degree program. Typical curriculum and instruction courses in math are 72 hours long, which must include any "math" topics to be examined

Ontario is in a time of conflict between a "back to basics" rhetoric, and research arguing for learning through inquiry and problem-solving. Textbooks are commercially developed and often include a lot of procedural skill focus given this conflict.

The work we describe here lends itself to exploration, but *can* also be handled in a more teacher-directed manner.

Push to “decolonize” mathematics

- In parallel, there is a significant push to do a better job in supporting Indigenous learners in mathematics learning
- All Canadian Indigenous languages are verb-based, with a focus on visual, concrete ideas, function and relationships (cite Lisa’s work)
- For example, in one language the word for square translates to “it sits flat”
- Many teachers, particularly those whose background is *mathematics*, rather than education or the social sciences, feel they do not have the background to support culturally-based learning contexts

Background to our work

- Began to survey incoming elementary teacher candidates in 2004 to determine the level of conceptual understanding of school mathematics.
- Overall extremely low level of depth of understanding, although in some cases they did recall the procedures

(e.g., Kajander. A. (2010). Mathematics teacher preparation in an era of curriculum change: The development of mathematics for teaching. *Canadian Journal of Education. 33*(1), 228-255.)

- Surprisingly low level of deep understanding also found in those with post-secondary mathematics background

(e.g., Holm, J. & Kajander, A. (2020). Seeking intersections: Math degrees, beliefs, and elementary teacher knowledge. *Canadian Journal of Science, Mathematics and Technology Education, 20*(1*),* 27-41. https://doi.org/10.1007/s42330-019-00069-3)

Our assumptions

The work to be shared today is based on the following assumptions

- All incoming teacher-candidates in our region, even those with math degrees, need to develop conceptual depth in the mathematics they will be teaching (e.g. “mathematics for teaching”)
- Such depth (often termed “specialized content knowledge” (Ball et al) in North America) directly supports student learning and achievement (e.g. Baumert et al, 2010)
- An important basis for conceptual understanding is the use of *representations and reasoning* to develop ideas
- Learning based on representations and reasoning satisfies many of the needs of Indigenous learning styles, *without* learning specifically about the cultures

Work on specialised content knowledge and use of representation to date in the field

- Mainly focused in North America on Grades 1-6
- Some work in Grades 7-8

(e.g. Kajander, A. and Boland, T. (2014). *Mathematical models for teaching: Reasoning without memorization.* Toronto, ON: Canadian Scholars' Press.)

- High school teachers here often have the perception that students have to be "told" the procedures first, before exploration. (Jenn? Cite something of yours?)

- **Even resources which claim to support secondary teachers' mathematics understanding often begin by *assuming* readers know the procedures and then *explaining* them**. -> Jennifer calls this "procedures with pictures"

To follow are two examples of our current work

- Developed for the intermediate and early secondary classrooms and currently in use with our teacher-candidates. Both explore alternatives to “telling” students procedures in typical high school algebra classes
- We acknowledge that less emphasis on procedural fluency might in fact be appropriate in new curricula, but it is still a mainstay requirement in our high school curriculum

Example 1 – simple factoring (used often in Ann’s own grade 9 classroom)

Example 2 – pushing the use of exploration based on representations and reasoning to grades 10 and 11 algebraic procedure development (used with preservice high school teachers)

Hands-On Exploration …

Thank you!

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Forthcoming:

Kajander, A. (2023). *Mathematics for intermediate teachers: From models to methods*. Cambridge Scholars Publishing: Great Britain.